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S-Matrix Approach to the Z Line Shape

A. Leike*, **T. Riemann***

Institut für Hochenergiephysik, Platanenallee 6
O-1615 Zeuthen, Germany

and

J. Rose*

III. Physikalisches Institut A der Rheinisch-Westfälischen Hochschule,
Sommerfeldstr. 26, W-5100 Aachen, Germany

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Abstract

We analyze the Z line shape assuming the existence of an analytic, unitary S-matrix. As an example, from hadron production at LEP we determine $M_Z = 91.134 \pm 0.020 \pm 0.020(\mathrm{LEP})$ GeV, $\Gamma_Z = 2.506 \pm 0.018$ GeV. This is in accordance with earlier results after performing a shift of the Z mass value of about $\frac{1}{2}\Gamma_Z^2/M_Z = 34$ MeV. The cross section and related observables may be described by a small number of additional degrees of freedom without relying on a specific field-theoretic model.

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In the early days of hadron physics, the contours of a satisfying dynamic theory were far from obvious. The analysis of resonance scattering had to be performed with a minimum of theoretical assumptions. Basics of the S-matrix theory which was developed in this context and its application to the description of resonances may be found e.g. in [1, 2, 3].

The present understanding of the gauge theory of electroweak interactions [4] allows for detailed and precise theoretical predictions of electroweak scattering including the precise measurements of Z boson interactions at LEP100. Nevertheless, it is of some interest to perform also model-independent fits to the Z line shape [5, 6]. In this respect, we consider the S-matrix theory to be the most consequent approach. An introduction to the necessary formalism and its application is the subject of the present article.

The annihilation of electrons and positrons into lepton pairs or hadrons at LEP100,

$$e^+e^- \longrightarrow (\gamma, Z) \longrightarrow f^+f^-(\gamma), \tag{1}$$

is used to determine mass M_Z and width Γ_Z of the Z boson. These observable quantities correspond uniquely to the location of a pole of the S-matrix describing (1) in the complex energy plane:

$$\mathcal{M}(s) = \frac{R_\gamma}{s} + \frac{R_Z}{s - s_Z} + F(s). \tag{2}$$

The poles of $\mathcal{M}$ have complex residua R_Z and R_γ, the latter corresponding to the photon, and $F(s)$ is an analytic function without poles. Further,

$$s_Z = M_Z^2 - iM_Z\Gamma_Z. \tag{3}$$

The analysis of the Z line shape will be based here on the cross section

$$\sigma(s) = \sum_{i=1}^{4} \sigma^i(s) = \frac{1}{4}\sum_{i=1}^{4} s|\mathcal{M}^i(s)|^2, \tag{4}$$

where the sum must be performed over four helicity amplitudes with different residua R_Z^i and functions $F^i(s)$ [1].

Although we will not perform a field theoretic interpretation here, for the reader's convenience the Born predictions of R_γ and R_Z in terms of vector and axial vector couplings are shown:

$$R_\gamma^B = \sqrt{\frac{4\pi}{3}c_f(1+\frac{\alpha_s}{\pi})}Q_eQ_f\alpha(s), \tag{5}$$

$$\begin{aligned}
R_Z^{0,B} &= R_Z(e_L^-e_R^+ \to f_L^-f_R^+) = c(v_e+a_e)(v_f+a_f), \\
R_Z^{1,B} &= R_Z(e_L^-e_R^+ \to f_R^-f_L^+) = c(v_e+a_e)(v_f-a_f), \\
R_Z^{2,B} &= R_Z(e_R^-e_L^+ \to f_R^-f_L^+) = c(v_e-a_e)(v_f-a_f), \\
R_Z^{3,B} &= R_Z(e_R^-e_L^+ \to f_L^-f_R^+) = c(v_e-a_e)(v_f+a_f).
\end{aligned} \tag{6}$$

[1] An application of the S-matrix formalism to e^+e^--annihilation has been proposed also in [7]. It is not pointed out there that one has to rely on helicity amplitudes and a simple-minded application of the formulae discussed there would fail.

In the standard theory,

$$c = \sqrt{\frac{4\pi}{3} c_f (1 + \frac{\alpha_s}{\pi}) \frac{G_\mu}{\sqrt{2}} \frac{M_Z^2}{2\pi}}, \qquad a_f = \pm\frac{1}{2}, \qquad v_f = a_f (1 - 4|Q_f| \sin^2 \vartheta_W), \tag{7}$$

where c_f is a possible color factor in case of hadron production. The corresponding functions $F^i(s)$ vanish in Born approximation, $F^{i,B}(s) = 0$. In general, the $F^i(s)$ contain non-resonating radiative corrections. More details on the realization of ansatz (2) in the standard theory may be found in [7].

Instead referring to field theory, we parametrize the cross section (4) as follows:

$$\sigma(s) = \sum_A \sigma_A(s), \qquad A = Z, \gamma, F, \gamma Z, ZF, F\gamma, \tag{8}$$

with the contributions:

$$\begin{aligned}
\sigma_Z(s) &= \frac{s r_Z}{|s - s_Z|^2}, & r_Z &= \tfrac{1}{4} \sum |R_Z^i|^2, \\
\sigma_\gamma(s) &= \frac{r_\gamma}{s}, & r_\gamma &= |R_\gamma|^2, \\
\sigma_F(s) &= s r_F(s), & r_F(s) &= \tfrac{1}{4} \sum |F^i(s)|^2, \\
\sigma_{\gamma Z}(s) &= 2\mathrm{Re} \frac{C_\gamma^* C_Z}{s - s_Z}, & C_\gamma &= R_\gamma, \quad C_Z = \tfrac{1}{4} \sum R_Z^i, \\
\sigma_{ZF}(s) &= 2\mathrm{Re} \frac{s C_{ZF}(s)}{s - s_Z}, & C_{ZF}(s) &= \tfrac{1}{4} \sum R_Z^i F^{i*}(s), \\
\sigma_{F\gamma}(s) &= 2\mathrm{Re} \left[C_\gamma^* C_F(s) \right], & C_F(s) &= \tfrac{1}{4} \sum F^i(s).
\end{aligned} \tag{9}$$

After making denominators real one remains with the following formula for the line shape:

$$\sigma(s) = \frac{R + (s - M_Z^2) I}{|s - s_Z|^2} + \frac{r_\gamma}{s} + r_0 + (s - M_Z^2) r_1 + \ldots \tag{10}$$

Besides M_Z, Γ_Z, the real constants R, I, r_0 and r_1 are introduced:

$$\begin{aligned}
R &= M_Z^2 \left[r_Z + 2(\Gamma_Z/M_Z) \left(\Im m C_R + M_Z \Gamma_Z \Re e(C_R') \right) \right], \\
I &= r_Z + 2\Re e C_R, \\
C_R(s) &= C_\gamma^* C_Z + s_Z C_{ZF}(s), \\
r_0 &= M_Z^2 \left[r_F - M_Z \Gamma_Z \Im m(r_F') \right] + \Re e C_r - M_Z \Gamma_Z \Im m C_r', \\
r_1 &= r_F + M_Z^2 \left[\Re e(r_F') - (\Gamma_Z/M_Z) \Im m(r_F') \right] + \Re e C_r', \\
C_r(s) &= C_\gamma^* C_F(s) + C_{ZF}(s).
\end{aligned} \tag{11}$$

The energy-dependent functions C_{ZF}, C_F, r_F, and their (primed) derivatives with respect to s have to be taken at $s = s_Z$. As may be seen, the cross section may be described by only six real parameters as long as one takes into account only the first two terms in the expansion of the functions $F^i(s)$ around $s = s_Z$ and at most terms of the order $(s - M_Z^2)^n, n = 0, 1$ in the cross section parametrization.

Next we have to discuss a conceptual problem due to QED bremsstrahlung. Initial state radiation of photons leads to a deviation of the *effective* energy variable s' in (4) from s with $s' < s$. At least events with soft photon emission unavoidably become part of the measured cross section. Taking them into account means summing up an infinitely dense chain of single poles leading to a new singularity structure (including a cut in the complex plane) compared to the original ansatz (4). Indeed, the well-known formulae for initial state radiation in reaction (1) (see e.g. [5, 8] and refs. therein) contain in one way or the other the complex logarithm

$$L(s_Z) = \ln \frac{1 - \Delta - s_Z/s}{1 - s_Z/s}, \tag{12}$$

where $0 < \Delta < 1$ stands for some cut on the allowed photon energies. A function like (12) with its highly singular behaviour at $s = s_Z$ cannot be absorbed into the function $F(s)$ as introduced in (2).

The QED bremsstrahlung must be treated as follows. Initial state radiation has to be taken into account as exactly as possible, e.g. using a convolution formula [5],

$$\sigma_T(s) = \int ds' \sigma(s') \rho_{\text{ini}}(1 - s'/s). \tag{13}$$

Final state radiation can be either calculated similarly or formally simply neglected. It doesn't influence the singularity structure of the cross section and leads to some modifications of parameters other than s_Z in (2) [2]. The radiation connected with initial-final state interferences can be taken into account by an analogue formula to (13) with a slightly more complicated structure [8, 9]:

$$\sigma_{\text{int}}(s) = \int ds' \sigma(s, s') \rho_{\text{int}}(1 - s'/s). \tag{14}$$

The correct ansatz for the S-matrix based cross section is:

$$\sigma(s, s') = \frac{1}{8} s' \sum_i \left[\mathcal{M}_i(s) \mathcal{M}_i^*(s') + \mathcal{M}_i^*(s) \mathcal{M}_i(s') \right]. \tag{15}$$

We only mention that a representation like (9) may be obtained easily also for $\sigma(s, s')$. If necessary, cross section (14) may be added to (13). Its numerical contribution is very small at LEP100 energies under usually applied cut conditions.

Using (2) - (4) for a fit to data, one is free of any model-dependent assumption, or some choice of gauge, or a truncation of perturbation theory as must be usually taken into account (see e.g. [5] and the recent discussion in [7, 10, 11]). The actual configuration of cuts applied to the data is as unimportant as are details of the final state. In case of a differential cross section, the S-matrix would depend on additional variables.

In order to demonstrate that the S-matrix approach may have some practical relevance, we use a simple code [12] for the calculation of the QED corrections (13) including soft photon exponentiation. We perform four fits with a rising number of degrees of freedom to published

[2] If one wants to interpret the residua R_γ and R_Z, and $F(s)$ in terms of a field theory, one should of course make an explicit calculation of final state radiation.

data at seven different beam energies E, $s = 4E^2$, taken from an analysis of the hadronic line shape (Table 2 of [13]). Without loss of generality, one can assume that the behaviour of the running coupling constant of the photon $\alpha(s)$ is known at LEP100 energies [14]:

$$\alpha\left(s = (91.2\text{GeV})^2\right) = \alpha_0(1.0660 - i0.0189). \tag{16}$$

For comparison, we give also the field theoretic Born estimates for hadron production:

$$\begin{aligned}
R_\gamma^{B,h} &= r^{1/2}\sqrt{\frac{4\pi}{3}c_f(1+\frac{\alpha_s}{\pi})}\alpha(s), \\
r_Z^{B,h} &= c^2(v_e^2+a_e^2)[3(v_d^2+a_d^2)+2(v_u^2+a_u^2)] \sim 6.32\ 10^{-4}, \\
C_Z^{B,h} &= \frac{c}{r^{1/2}}Q_e v_e[3Q_d v_d + 2Q_u v_u] \sim 7.77\ 10^{-4}, \\
r &= 3Q_d^2 + 2Q_u^2, \\
C_{ZF}^{B,h} &= C_F^{B,h} = r_F^{B,h} = 0.
\end{aligned} \tag{17}$$

Further,

$$\begin{aligned}
R^{B,h} &= M_Z^2 r_Z^{B,h} &&= 5.45\ \text{GeV}^2, \\
I^{B,h} &= r_Z^{B,h} + 2(\Re e C_\gamma^*)C_Z &&= 7.05\times 10^{-3}, \\
r_0^{B,h} &= 0.0\ \text{GeV}^{-2}, \\
r_1^{B,h} &= 0.0\ \text{GeV}^{-4}.
\end{aligned} \tag{18}$$

These numerical estimates are obtained with the weak parameters quoted in [13].

In our first fit with four free parameters we fix all quantities which are zero in the Born approximation. Then we allow for additional parameters (r_0, r_1) to be fitted. The numerical results are shown in Table 1. The small number of available data points is certainly disadvantageous for the fit results. Nevertheless, the table gives some impression on the potential value of the approach. The gain of accuracy for a smaller number of floating parameters is a measure of the degree of biasing the fits with certain assumptions usually done in a specific ansatz. From our starting point it is evident what would be a completely unbiased fit - taking into account all higher powers of $(s - M_Z^2)$ in the cross section ansatz and of $(s - s_Z)$ in the Taylor expansions of the $F^i(s)$.

As is known from earlier fits, the determination of the residuum and of the non-resonating terms is not too stringent if one analyzes only the line shape. While our accuracy for the Z width is comparable to other determinations, we have a larger error for the mass. This is due to a strong correlation between M_Z and the parameters I, r_1 in (10), which are not fixed here from the beginning. If one would assume them to be known from other sources, the mass determination would be better. Similarly, the Z width is correlated with R, r_0. The smaller error of R compared to that of I leads to the relatively small error of Γ_Z compared to that of M_Z.

The measured Z mass value differs from earlier determinations by a non-negligible shift. This is an immediate consequence of the S-matrix approach. The parametrization of the Breit-Wigner resonance formula for the Z peak as being inspired by perturbation theory assumes usually (but *not necessarily* [15, 16]) an s-dependent width function $\bar{\Gamma}_Z(s)$ [5, 13, 17, 18, 19, 20].

nb. of parameters	4	5	5	6
M_Z Γ_Z	91.134±.020 2.506±.018	91.130±.020 2.484±.040	91.120±.032 2.490±.034	91.128±.046 2.484±.041
R, GeV^2 $I \times 10^3$	5.49±0.08 8.9±2.4	5.38±0.21 9.5±2.5	5.41±0.17 12.1±6.3	5.38±0.21 10.1±11.3
$r_0 \times 10^7, \mathrm{GeV}^{-2}$ $r_1 \times 10^{10}, \mathrm{GeV}^{-4}$	– –	3.5±5.9 –	– –5.6±10.5	3.0±13.1 –1.3±23.

Table 1: Results of S-matrix based fits to the hadronic line shape as measured at LEP100. An uncertainty of 20 MeV in the energy scale of LEP must yet be added to the error of M_Z.

In our notations, this would correspond to the following ansatz:

$$\mathcal{M}^i(s) = \frac{R_\gamma}{s} + \frac{\bar{R}_Z^i}{s - \bar{s}_Z(s)} + \bar{F}^i(s),$$
$$\bar{s}_Z(s) = \bar{M}_Z^2 - i\bar{M}_Z\bar{\Gamma}_Z(s). \tag{19}$$

The difference between M_Z, Γ_Z, R_Z^i and $\bar{M}_Z, \bar{\Gamma}_Z, \bar{R}_Z^i$ is described by a transformation proposed earlier in another context [21]:

$$\bar{M}_Z = M_Z\sqrt{1 + \Gamma_Z^2/M_Z^2} \approx M_Z + \frac{1}{2}\Gamma_Z^2/M_Z = M_Z + 34\mathrm{MeV},$$

$$\bar{\Gamma}_Z = \Gamma_Z\sqrt{1 + \Gamma_Z^2/M_Z^2} \approx \Gamma_Z + \frac{1}{2}\Gamma_Z^3/M_Z^2 = \Gamma_Z + 1\mathrm{MeV},$$

$$\bar{R}_Z = R_Z(1 + i\Gamma_Z/M_Z). \tag{20}$$

This transformation is exact as long as there are no thresholds (opening new decay channels) or rapidly changing radiative corrections in the vicinity of the Z peak position. Then,

$$\bar{\Gamma}_Z(s) = \frac{s}{\bar{M}_Z^2}\bar{\Gamma}_Z. \tag{21}$$

If (21) would be exact, it would follow $\bar{F}(s) = F(s)$. A dependence of mass and width determinations on the theoretical ansatz for a line shape description has been observed earlier, see e.g. [2]. There, formulae (20) may also be applied in order to relate different approaches to the hadron resonances under discussion. We further mention that the ratio of width and mass is invariant:

$$\frac{\bar{\Gamma}_Z}{\bar{M}_Z} = \frac{\Gamma_Z}{M_Z}. \tag{22}$$

Although we think that the natural application of the S-matrix approach to Z boson physics is the line shape analysis, we indicate also how other observables than $\sigma_T(s)$ may be treated. For a calculation of e.g. the initial state bremsstrahlung contribution to the left-right asymmetry A_{LR}, the convolution for the numerator $\sigma_{LR}(s)$ must be performed with a modified ansatz:

$$A_{LR} = \frac{\sigma_{LR}(s)}{\sigma_T(s)}, \qquad \sigma_{LR}(s) = \int ds' \left[\sigma_0 + \sigma_1 - \sigma_2 - \sigma_3\right](s')\rho_{\rm ini}(1 - s'/s). \tag{23}$$

The helicity cross sections σ_i are introduced in (4). Similarly, the final state polarisation A_{pol} may be obtained:

$$A_{pol} = \frac{\sigma_{pol}(s)}{\sigma_T(s)}, \qquad \sigma_{pol}(s) = \int ds' \left[\sigma_0 - \sigma_1 - \sigma_2 + \sigma_3\right](s')\rho_{\rm ini}(1 - s'/s). \tag{24}$$

The forward-backward asymmetry A_{FB} deserves additional comments. Due to the different angular integrations, the weight functions (flux factors) $R_{\rm ini}(z)$ etc. of the forward-backward difference cross section $\sigma_{FB}(s)$ differ from those for $\sigma_T(s)$. Nevertheless, a convolution may be derived [8, 9, 22]:

$$A_{FB} = \frac{\sigma_{FB}(s)}{\sigma_T(s)}, \qquad \sigma_{FB}(s) = \int ds' \left[\sigma_0 - \sigma_1 + \sigma_2 - \sigma_3\right](s')R_{\rm ini}(1 - s'/s). \tag{25}$$

Strictly speaking, the σ_i for the forward-backward asymmetry are different from those for the total cross section. They agree in Born approximation. Further, we know from one-loop calculations in the standard theory which changes are to be expected after the introduction of e.g. weak form factors as proposed in [23].

A study of the usefulness of these formulae should be performed with a more sophisticated code than ZPOLE in order to describe more realistic cut situations. A modified version of ZFITTER [14] for this purpose is in preparation. A larger number of experimental data points would also be highly desirable due to the large number of degrees of freedom of the line shape.

To summarize, we formulate an alternative approach to the Z line shape assuming the existence of an analytic, unitary S-matrix and the validity of QED.

We demonstrate that the approach allows reasonable fit results for mass and width of the Z boson. The Z mass obtained this way differs by a well-understood shift of -34 MeV from earlier measurements. The other line shape parameters determined by the fit can also be interpreted e.g. in the standard theory.

A dedicated application of the S-matrix approach to polarized scattering and to Z' physics would also be interesting.

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